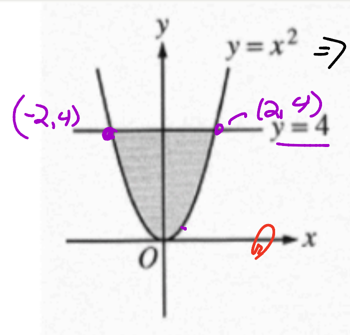
Period 3, April 28, 2025




$\Rightarrow x = \sqrt{y}$

1. The shaded region, $R$, is bounded by the graph of $y = x^2$ and the line $y = 4$, as shown in the figure above.

   (a) Find the area of $R$.

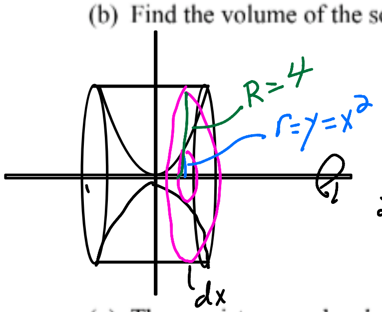
$$\int_{-2}^{2} [4 - x^2]\,dx = 2\int_{0}^{2} [4 - x^2]\,dx = 2\int_0^4 \sqrt{y}\,dy$$

$$2\left(\frac{16}{3}\right) = \frac{32}{3}$$

$$4x - \frac{1}{3}x^3 \Big|_0^2 = 4(2) - \frac{1}{3}(2)^3 - \cancel{\left[4(0) - \frac{1}{3}(0)^3\right]}$$

$$8 - \frac{8}{3} = \frac{24-8}{3} = \frac{16}{3}$$

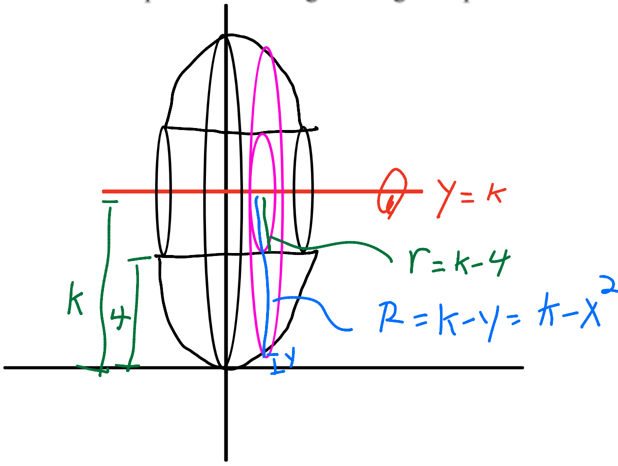
   (b) Find the volume of the solid generated by revolving $R$ about the $x$-axis.



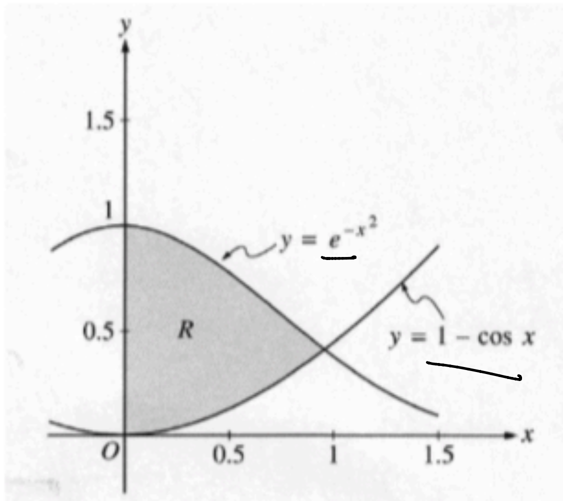

$$\pi\int_{-2}^{2} [R^2 - r^2]\,dx = 2\pi\int_0^2 [R^2 - r^2]\,dx$$

$$2\pi\int_0^2 [4^2 - (x^2)^2]\,dx = 2\pi\int_0^2 (16 - x^4)\,dx = 2\pi\left[16x - \frac{x^5}{5}\right]\Big|_0^2 = \frac{256\pi}{5}$$

   (c) There exists a number $k$, $k > 4$, such that when $R$ is revolved about the line $y = k$, the resulting solid has the same volume as the solid in part (b). Write, but do not solve, an equation involving an integral expression that can be used to find the value of $k$.




$$\pi\int_{-2}^{2} \left[(k - x^2)^2 - (k-4)^2\right]dx$$



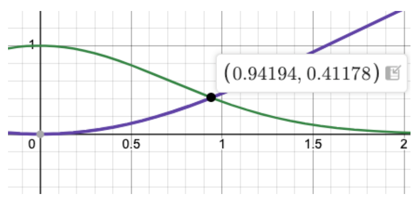

2. Let $R$ be the shaded region in the first quadrant enclosed by the graphs of $y = e^{-x^2}$, $y = 1 - \cos x$ and the $y$-axis, as shown in the figure above.

$x \approx 0.94194$

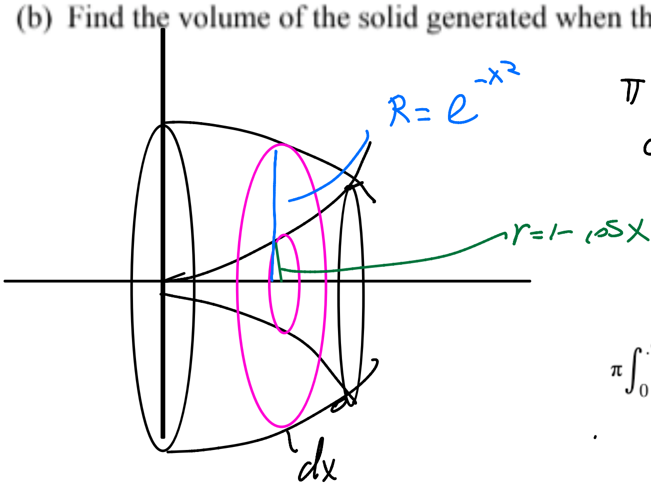
(a) Find the area of the region $R$.

$$\int_0^{0.94} \left[ e^{-x^2} - (1 - \cos x) \right] dx$$

$$\int_0^{.94194} \left[ e^{-x^2} - (1 - \cos x) \right] dx = 0.590962450123$$




(b) Find the volume of the solid generated when the region $R$ is revolved about the $x$-axis.




$$\pi \int_0^{.94194} (R^2 - r^2)\, dx$$

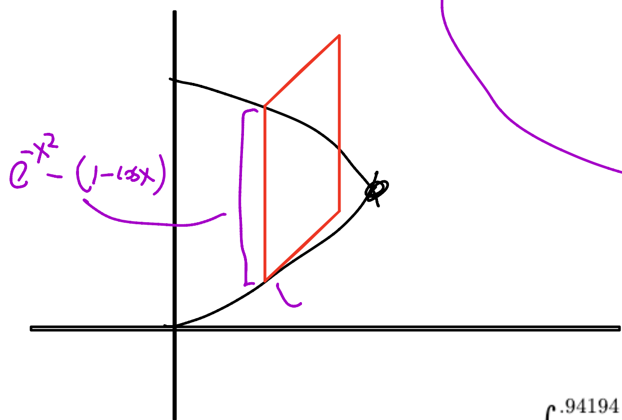
$$\pi \int_0^{0.94194} \left[ (e^{-x^2})^2 - (1 - \cos x)^2 \right] dx$$

$$\pi \int_0^{.94194} \left[ \left(e^{-x^2}\right)^2 - (1 - \cos x)^2 \right] dx = 1.74661409822$$

(c) The region $R$ is the base of a solid. For this solid, each cross section perpendicular to the $x$-axis is a square. Find the volume of this solid.

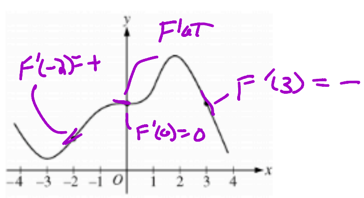
$dx$ $\qquad$ $A = (\text{Length of side})^2$

$$\int_0^{0.94194} \left[ e^{-x^2} - (1-\cos x) \right]^2 dx$$




$$\int_0^{.94194} \left[ \left(e^{-x^2}\right) - (1-\cos x) \right]^2 dx$$

$= 0.461063510705$



Graph of $f$

The graph of a differentiable function $f$ is shown in the figure above.

(A) $f'(-2) < f'(0) < f'(3)$

(B) $f'(-2) < f'(3) < f'(0)$

(C) $f'(3) < f'(-2) < f'(0)$

$- < 0 < +$

(D) $f'(3) < f'(0) < f'(-2)$

Let $H(x)$ be an antiderivative of $\frac{x^3+\sin x}{x^2+2}$. If $H(5) = \pi$, then $H(2) =$

$$\int_5^2 \left[\frac{x^3+\sin x}{x^2+2}\right]dx = -9.00825700345$$

Change in H(x)

From 5 To 2

Start at X=5

Y=π

H(2)=π−9.00826

Ⓐ −9.008

Ⓑ −5.867

Ⓒ 4.626

Ⓓ 12.150

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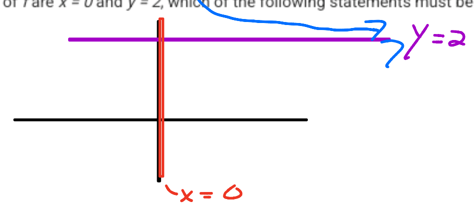
$\lim_{x\to a} F(x)$ exists

F(a) Exists

$F(a) = \lim_{x\to a} F(x)$

Mr. Bill can walk

The continuous function $f$ is positive and has domain $x > 0$. If the asymptotes of the graph of $f$ are $x = 0$ and $y = 2$, which of the following statements must be true?




Ⓐ $\lim_{x\to 0^+} f(x) = \infty$ and $\lim_{x\to 2} f(x) = \infty$ 0

Ⓑ $\lim_{x\to 0^+} f(x) = 2$ and $\lim_{x\to \infty} f(x) = 0$ 0

Ⓒ $\lim_{x\to 0^+} f(x) = \infty$ and $\lim_{x\to \infty} f(x) = 2$ ✔ 1

Ⓓ $\lim_{x\to 2} f(x) = \infty$ and $\lim_{x\to \infty} f(x) = 2$ 0

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Rate of download

A file is downloaded to a computer at a rate modeled by the differentiable function $f(t)$, where $t$ is the time in seconds since the start of the download and $f(t)$ is measured in megabits per second. Which of the following is the best interpretation of $f'(5) = 2.8$ ?

F'(5) = Rate of Change of Rate of Download

Ⓐ At time $t = 5$ seconds, the rate at which the file is downloaded to the computer is 2.8 megabits per second. (B) (C)(D) 0

Ⓑ At time $t = 5$ seconds, the rate at which the file is downloaded to the computer is increasing at a rate of 2.8 megabits per second per second. ✔ 0

Ⓒ Over the time interval $0 \le t \le 5$ seconds, 2.8 megabits of the file are downloaded to the computer. 0

Ⓓ Over the time interval $0 \le t \le 5$ seconds, the average rate at which the file is downloaded to the computer is 2.8 megabits per second. 0

$F''(x) = +$

The function $f$ has first derivative given by $f'(x) = x^4 - 6x^2 - 8x - 3$. On what intervals is the graph of $f$ concave up?

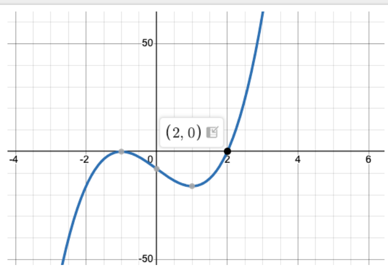
$F''(x) = 4x^3 - 12x - 8$

Ⓐ $(2, \infty)$ only ← $x > 2$

Ⓑ $(0, \infty)$

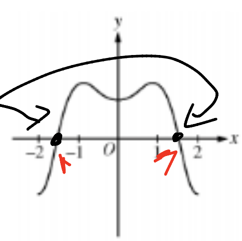
Ⓒ $(-1, 2)$

Ⓓ $(-\infty, -1)$ and $(3, \infty)$

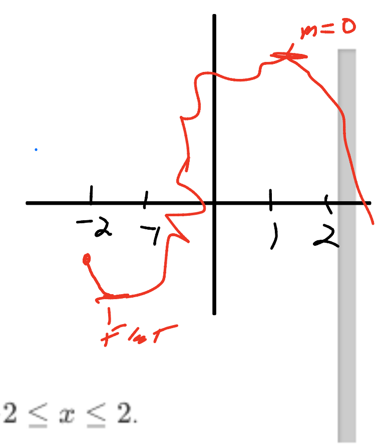



$g(x) = \int F(x)\,dx$

$g'(x) = 0$ → Flat parts of $g(x)$



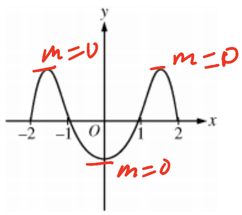

Graph of $f$



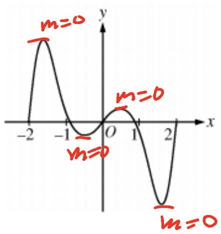

The graph of the function $f$ is shown above for $-2 \le x \le 2$.

Which of the following could be the graph of an antiderivative of $f$? - area under curve

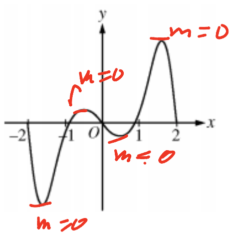
Ⓐ



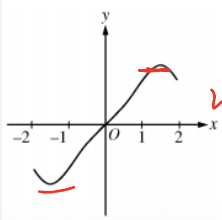

Ⓑ




Ⓒ




Ⓓ




with 2 m=0

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$f''(x) = x(x-1)^2(x+2)^3$

$g''(x) = x(x-1)^2(x+2)^3 + 1$

$h''(x) = x(x-1)^2(x+2)^3 - 1$

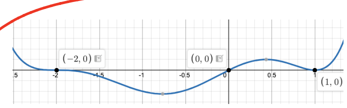
The twice-differentiable functions *f*, *g*, and *h* have second derivatives given above. Which of the functions *f*, *g*, and *h* have a graph with exactly two points of inflection?

Ⓐ *g* only

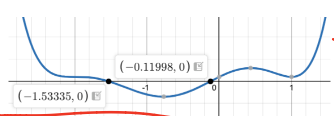
Ⓑ *h* only

Ⓒ *f* and *g* only

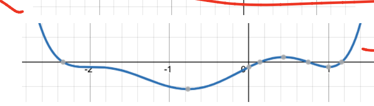
Ⓓ *f*, *g*, and *h*




− f''(x) 2




− f''(x) 2



− h''(x) 4

If $f$ is twice-differentiable on the interval $1 \leq x \leq 5$, which of the following statements could be true?

$$\frac{F(5)-F(1)}{5-1} = \frac{-5-9}{4} = \frac{-14}{4}$$

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $f(x)$ | 9 | 4 | 0 | –3 | –5 |

$m=-5$ $m=-4$ $m=-3$ $m=-2$

Slope is increasing

The table above gives values of a function $f$ at selected values of $x$.

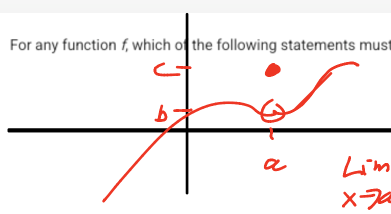
(A) $f'$ is negative and decreasing for $1 \leq x \leq 5$.

(B) $f'$ is negative and increasing for $1 \leq x \leq 5$.

(C) ~~$f'$ is positive and decreasing for $1 \leq x \leq 5$.~~

(D) ~~$f'$ is positive and increasing for $1 \leq x \leq 5$.~~

For any function $f$, which of the following statements must be true?




$\lim_{x \to a} F(x) = b$

$F(a) = c$

I. ~~If $f$ is defined at $x = a$, then $\lim_{x \to a} f(x) = f(a)$.~~

II. If $f$ is continuous at $x = a$, then $\lim_{x \to a} f(x) = f(a)$.

III. If $f$ is differentiable at $x = a$, then $\lim_{x \to a} f(x) = f(a)$.

also continious

$\lim_{x \to a} F(x)$ Exists

$F(a)$ Exists

$F(a) = \lim_{x \to a} F(x)$

(A) III only

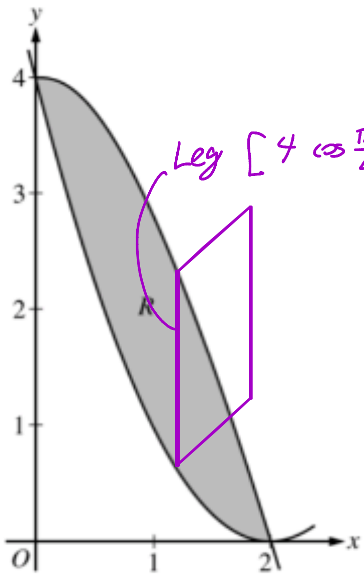
(B) I and II only

(C) II and III only

(D) I, II, and III

$$\int_0^2 \frac{1}{2}\left(4\cos\frac{\pi x}{4}-(x-2)^2\right)^2 dx$$

Leg $\left[4\cos\frac{\pi x}{4}-(x-2)^2\right]$




Let $R$ be the region in the first quadrant bounded by the graphs of $y = 4\cos\left(\frac{\pi x}{4}\right)$ and $y = (x-2)^2$, as shown in the figure above. The region $R$ is the base of a solid. For the solid, each cross section perpendicular to the $x$-axis is an isosceles right triangle with a leg in region $R$.

$dx$

$\frac{1}{2}(\text{Leg})^2$

(A) 1.775

$$\frac{1}{2}\int_0^2\left[4\cos\left(\frac{\pi x}{4}\right)-(x-2)^2\right]^2 dx$$

$= 1.77456261736$

(B) 3.549

(C) 4.800

(D) 5.575

Let $f$ be a twice-differentiable function such that $f''(x) < 0$ for all $x$. The graph of $y = S(x)$ is the secant line passing through the points $(3, f(3))$ and $(5, f(5))$. The graph of $y = T(x)$ is the line tangent to the graph of $f$ at $x = 4$.

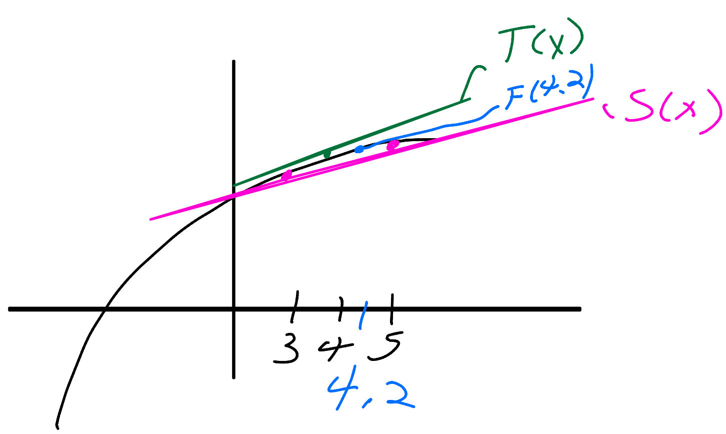
Which of the following is true?

Concave Down

Ⓐ $f(4.2) < S(4.2) < T(4.2)$

Ⓑ $f(4.2) < T(4.2) < S(4.2)$

Ⓒ $S(4.2) < f(4.2) < T(4.2)$




$S(4.2) < F(4.2) < T(4.2)$

If $g$ is a differentiable function such that $h(g(x)) = x$ for all $x$, what is the value of $g'(7)$ ?

$$g'(x) = (h^{-1})'(x) = \frac{1}{h'(h^{-1}(x))}$$

$$g'(7) = \frac{1}{h'(g(7))}$$

$$\frac{1}{h'(3)} = \frac{1}{5}$$

$h(g(x)) = x$

$\cancel{h^{-1}(h}(g(x)) = h^{-1}(x)$

$g(x) = h^{-1}(x)$

$g'(7) = (h^{-1})'(7)$

$g(7) = h^{-1}(7)$

$h^{-1}(7) = a$

$h(a) = 7$

$a = 3$

$h(3) = 7$

$h^{-1}(7) = 3$

$g(7) = 3$

| $x$ | 3 | 7 |
|---|---|---|
| $h(x)$ | 7 | 22 |
| $h'(x)$ | 5 | 10 |

Selected values of the increasing function $h$ and its derivative $h'$ are shown in the table above.

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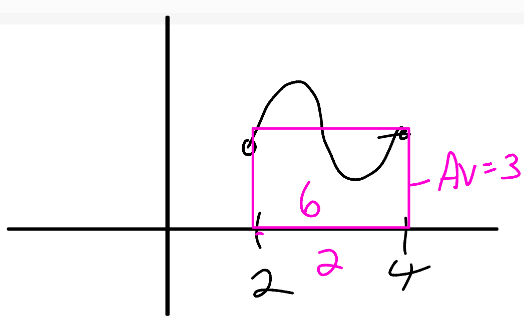
If the average value of the function $f$ over the closed interval $[2, 4]$ is 3 and if $f(x) \geq 0$ for all $x$ in $[2, 4]$, what is the area of the region enclosed by the graph of $y = f(x)$, the lines $x = 2$ and $x = 4$, and the x-axis?

(A) 12 — 0

(B) 6 — ✔ 7

(C) 3 — 0

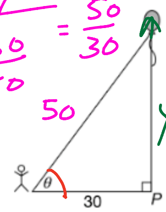
(D) 3/2 — 1




$$6 = \int_2^4 F(x)\,dx = \text{Area under curve}$$

Average = height of Rectangle

$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{30}{50}} = \frac{50}{30}$




$2\,FT/s$

$50$

$y = 40$

A person stands 30 feet from point $P$ and watches a balloon rise vertically from the point, as shown in the figure above. The balloon is rising at a constant rate of 2 feet per second.

$\frac{dy}{dT} = 2\,FT/s$

What is the rate of change, in radians per second, of angle $\theta$ at the instant when the balloon is 40 feet above point $P$?

$\frac{d\theta}{dT} = ?$

$\tan\theta = \frac{y}{30}$

$\sec^2\theta \frac{d\theta}{dT} = \frac{1}{30} \cdot \frac{dy}{dT}$

$\left(\frac{50}{30}\right)^2 \frac{d\theta}{dT} = \frac{1}{30} \cdot 2\,FT/sec$

$\left(\frac{5}{3}\right)^2 \frac{d\theta}{dT} = \frac{2}{30} = \frac{1}{15}$

$\cancel{\frac{9}{25}} \cdot \frac{25}{9} \frac{d\theta}{dT} = \frac{1}{15} \cdot \frac{9}{25}$

$\frac{d\theta}{dT} = \frac{1}{\cancel{15}_5} \cdot \frac{\cancel{9}^3}{25} = \frac{3}{125}\ R/s$

$\frac{a}{c}$

How many vertical asymptotes does the graph of $y = \frac{x-2}{x^4-16}$ have?

(A) One

(B) Two

(C) Three

(D) Four

$\frac{x-2}{x^4-16} = \frac{(x-2)}{(x^2-4)(x^2+4)}$

$\frac{\cancel{(x-2)}}{\cancel{(x-2)}(x+2)(x^2+4)}$

always +

$x+2=0$

$x=-2$

Always +

For what value of $b$ does the integral $\int_1^{b=3} x^2 dx$ equal $\lim_{n\to\infty} \sum_{k=1}^{n} \left(1+\frac{2k}{n}\right)^2 \underbrace{\frac{2}{n}}_{dx}$? 2 Length of interval

$1^2=1$
$3^2=9$

$k=1$ $n=\infty$
$\left(1+\frac{2}{\infty}\right)^2=(1+0)^2=1$

$k=n$ $n=\infty$
$\left(1+\frac{2n}{n}\right)^2=(1+2)^2=3^2=9$

(A) $b=2$ only

(B) $b=3$ only

(C) $b$ could be any real number.

(D) There is no such value of $b$.

**Question 3**

The wind chill is the temperature, in degrees Fahrenheit (°F), a human feels based on the air temperature, in degrees Fahrenheit, and the wind velocity $v$, in miles per hour (mph). If the air temperature is 32°F, then the wind chill is given by $W(v) = 55.6 - 22.1v^{0.16}$ and is valid for $5 \le v \le 60$.

(a) Find $W'(20)$. Using correct units, explain the meaning of $W'(20)$ in terms of the wind chill.

(b) Find the average rate of change of $W$ over the interval $5 \le v \le 60$. Find the value of $v$ at which the instantaneous rate of change of $W$ is equal to the average rate of change of $W$ over the interval $5 \le v \le 60$.

(c) Over the time interval $0 \le t \le 4$ hours, the air temperature is a constant 32°F. At time $t = 0$, the wind velocity is $v = 20$ mph. If the wind velocity increases at a constant rate of 5 mph per hour, what is the rate of change of the wind chill with respect to time at $t = 3$ hours? Indicate units of measure.

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$W(v)=55.6-22.1V^{0.16}$

$W'(v)=0-22.1(0.16)V^{0.16-1}$

$W'(v)=-3.536V^{-.84}=\frac{-3.536}{V^{.84}}$

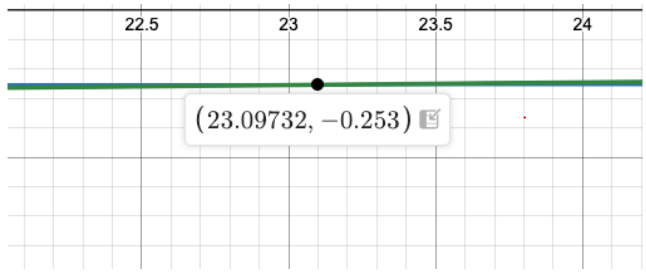
$W'(20)=\frac{-3.536}{(20)^{.84}}=\frac{-3.536}{12.38412}=-0.2855269$ degree/mph

(b) Find the average rate of change of $W$ over the interval $5 \le v \le 60$. Find the value of $v$ at which the instantaneous rate of change of $W$ is equal to the average rate of change of $W$ over the interval $5 \le v \le 60$.

Average Rate of Change = Slope between 5 and 60

$$\frac{w(60)-w(5)}{60-5} = \frac{w(60)-w(5)}{55} = -.253$$

$W'(v) = -0.253$




(c) Over the time interval $0 \le t \le 4$ hours, the air temperature is a constant 32°F. At time $t = 0$, the wind velocity is $v = 20$ mph. If the wind velocity increases at a constant rate of 5 mph per hour, what is the rate of change of the wind chill with respect to time at $t = 3$ hours? Indicate units of measure.

$\frac{dw}{dT}$ at $T=3 \Rightarrow V = 5 \cdot 3 + 20 = 35$ $\frac{dv}{dT}$

$$\frac{dw}{dV} = \frac{-3.536}{V^{.84}}$$

$$\frac{\cancel{dv}}{dT} \cdot \frac{dw}{\cancel{dV}}$$

$$5 \cdot \frac{-3.536}{V^{.84}} = \frac{dw}{dT} = \frac{\text{wind chill}}{\text{Time}}$$

$$5 \cdot \frac{-3.536}{35^{.84}} = -.8922059 \ {}^{\circ}F/h$$